QP CODE: 25050454

25050454

Reg No : .....

Name : .....

MSc DEGREE (CSS)EXAMINATION DECEMBER 2025

First Semester

M Sc MATHEMATICS

Core Courses - ME010103 - BASIC TOPOLOGY

2019 Admission Onwards

8DA3B197

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define convergence of a sequences in a topological space. Discuss the convergence of sequence in a discrete space.
2. Define order topology.
3. Define hereditary property Show that subspace of an indiscrete space is indiscrete.
4. Define neighborhood of a point and interior point of a set.
5. Define (i) homeomorphism (ii)imbedding
6. Define quotient topology and quotient space.
7. Let A be a clopen subset of a space X .Prove that boundary of A is empty.
8. Let $\mathscr{C}$ be a collection of connected subsets of a space X and suppose K is a connected subset of X such that $C \cap K \neq \emptyset, \forall C \in \mathscr{C}$. Prove that $\left(\cup_{C \in \mathscr{C}} C\right) \cup K$ is connected.
9. *Define T_0 and T_1 space with example. Give an example of a T_0 space which is not T_1*
10. *Define T_5 space*

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. What are the properties of open sets in a metric space?
12. Given any family S of subsets of X show that there exist a unique topology having S as a sub base
13. Let the function $f : X \to Y$ is $\mathfrak{I} - \mathcal{U}$ continuous, where $\mathfrak{I}, \mathcal{U}$ be topologies on X, Y respectively. Then show that for all $V \in \mathcal{U}$, $f^{-1}(V) \in \mathfrak{I}$ *if and only if* there exist a sub-base $\mathcal{S}$ for $\mathcal{U}$ such that $f^{-1}(V) \in \mathfrak{I}$, for all $V \in \mathcal{S}$

14. Let X have the strong topology determined by a family $\{f_i : Y_i \to X / i \in I\}$ of functions where each Y_i is a topological space, I being an index set. Then show that for any space Z, a function $g : X \to Z$ is continuous if and only if for each $i \in I$, the composite $g \circ f_i : Y_i \to Z$ is continuous.

15. Prove that every continuous real valued function on a compact space is bounded .

16. Prove or disprove : Every first countable space is second countable.

17. Show that path connectedness is preserved under continuous onto functions.

18. What is the peculiarity of limit of a converging sequence in a Hausdorff space.What is the converse.? Justify your answer.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Explain three equivalent conditions for a base of a topological space

20. Let A, B be subsets of a topological space X and $c(A)$ is the closure of A. Then show that (i) $c(A)$ is a closed set in X and it is the smallest closed subset of Xcontaining A. (ii) A is closed in X if and only if $c(A) = A$. (iii) $c(\phi) = \phi$. (iv) $c(c(A)) = c(A)$. (iv) $c(A \cup B) = C(A) \cup C(B)$.

21. (i) Let (X, τ) be a topological space and A be a subset of X. Prove that A is a compact subset of X if and onlyif the space $(A, \tau_{|A})$ is compact.
(ii) Prove that every second countable space is separable.

22. What are the equivalent conditions of locally connected space? Explain.

(2×5=10 weightage)